Operational space robot control for motion performance and safe interaction under Unintentional Contacts

Yiannis Karayiannidis[1], Leonidas Droukas[2] and Zoe Doulgeri[2,3]

***Abstract*— A control law achieving motion performance of quality and compliant reaction to unintended contacts for robot manipulators is proposed in this work. It achieves prescribed performance evolution of the position error under disturbance forces up to a tunable level of magnitude. Beyond this level, it deviates from the desired trajectory complying to what is now interpreted as unintentional contact force, thus achieving enhanced safety by decreasing interaction forces. The controller is a passivity model based controller utilizing an artificial potential that induces vanishing vector fields. Simulation results with a three degrees of freedom (DOF) robot under the control of the proposed scheme, verify theoretical findings and illustrate motion performance and compliance under an external force of short duration in comparison with a switched impedance scheme.**

I. INTRODUCTION

In the context of service robots, human safety is of primary importance. In human robot co-existence, the possibility of an unintentional contact between the robot and the human or the environment cannot be overruled by the existence of collision avoidance mechanisms; hence, unintentional contacts should still be accounted for and harm of collisions should be minimized. Compliance protects the human during unintentional contact whether abrupt or incipient, and can be achieved in robots either passively by using flexible components in the robot's structure or actively by the controller. Passive compliance is very important for the reduction of the initial collision force and may be achieved by using deformable material to cover the robot or by building robots with compliant or variable stiffness joints [1]–[5]. When mechanical compliance is absent, active compliance takes the responsibility of keeping contacts harmless. As controllers introducing active compliance at safety level are characterized by poor performance, contact detection and reaction strategies are proposed. Once a collision is detected the robot switches from the control law of its nominal task which is characterized by performance quality and hence high stiffness, to that of a reaction control law characterized by high compliance. However, a delay is usually introduced in the control system by the contact detection and reaction mechanisms. Control switching may be another source of delay and in general, may adversely affect the stability of the overall switched system. The residual torque method is a robot model based contact detection utilizing proprioceptive sensors and one or more external RGB-D sensors to localize the contact point, [6]–[8]. The utilization of a disturbance observer is proposed in a frequency shaped impedance control scheme in [9]. This is however mainly addressed to physical human robot interaction applications which are characterized by intentional contacts. The general problem of discriminating contacts to intentional and unintentional is examined in [10] where a machine learning method combined with features of physical contact models is proposed. Non-linear stiffness terms are introduced in impedance controllers for facilitating physical human robot interaction by setting different stiffness values in relation to deviation sizes around a nominal trajectory, [11]. For robots with variable stiffness actuated joints, the bandwidth of the stiffness actuating system is crucial for responding promptly to unexpected contacts.

The aim of this work is to concurrently address the competing requirements of motion performance and compliance under unintentional contact by designing a control scheme that self-regulates the control effort according to the disturbance level without explicit collision detection and control switching. The proposed controller achieves quality of performance in nominal operation and compliant reaction at unintentional contact assuming availability of joint position and velocity measurements and knowledge of the robot's model .

II. CONTROL PRELIMINARIES

Consider a first-order integrator scalar system of a tracking error e under disturbance $d(t)$:

$$\dot{e} = u + d(t), \text{ with } |d(t)| \leq \Delta, \forall t \ , \tag{1}$$

where u is the control input. This system can be viewed as a robotic degree of freedom controlled kinematically at the velocity level. We shall utilize this system to define operation modes and introduce the basic control idea.

A. *Operation Modes*

For system (1) we define two modes of system operation: the *nominal operation* and *contact reaction*.

[1]Y. Karayiannidis is with the Dept. of Signals and Systems, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden and with the Center for Autonomous Systems, Royal Institute of Technology (KTH), SE-100 44 Stockholm, Sweden. e-mail: yiannis@chalmers.se yiankar@kth.se

[2]L.Droukas and Z.Doulgeri are with Aristotle University of Thessaloniki, Department of Electrical and Computer Engineering,Thessaloniki 54124, Greece. e-mail: ldroukas@auth.gr , doulgeri@eng.auth.gr

[3]Z.Doulgeri is also Academic Research Associate at Center for Research and Technology Hellas (CERTH), 57001 Thessaloniki, Greece. e-mail: doulgeri@iti.gr

This work is partially funded by the EU Horizon 2020 research and innovation programme under grant agreement No 643433, project RAMCIP and the Swedish Research Council (VR).

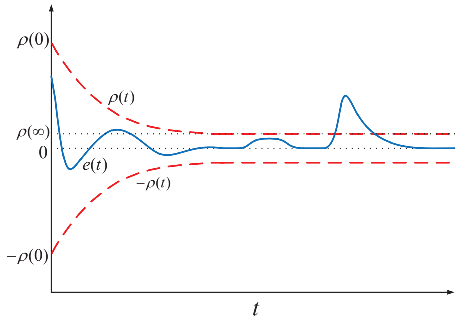

Fig. 1: Nominal operation and contact reaction mode.

The system in its nominal operation is desired to follow a position trajectory with quality of performance and in case of unintentional contact, it is desired to react compliantly without the use of any contact detection or control switching mechanisms. In the latter case, we say the system is in contact reaction mode of operation. To rigorously define the two modes of operation we make use of the prescribed performance concept utilized for designing robot motion controllers guaranteeing prescribed performance for the output error [12], [13]. More specifically, the system is in its nominal operation mode when motion performance in the sense of (2) is satisfied; that is, if the tracking error $e(t)$ evolves strictly within a predefined region that is bounded by a decaying function of time constructed by the designer:

$$-\rho(t) < e(t) < \rho(t) \ , \ \forall t \geq 0 \tag{2}$$

where $\rho(t)$ is a bounded, smooth, strictly positive and decreasing function satisfying $\lim_{t\to\infty} \rho(t) = \rho_\infty > 0$ called performance function. A candidate performance function is the exponential

$$\rho(t) = (\rho_0 - \rho_\infty) \exp(-lt) + \rho_\infty \tag{3}$$

with ρ_0, ρ_∞, l strictly positive constants expressing nominal performance specifications. Constant $\rho_0 = \rho(0) > e(0)$ while constant ρ_∞ represents the maximum allowable size of the output error $e(t)$ at steady state. Furthermore, constant l is related to the decreasing rate of $\rho(t)$ introducing a lower bound on the required speed of convergence of $e(t)$.

Furthermore, consider the modulated error $x = \frac{e(t)}{\rho(t)}$ and define the system in its nominal performance operation if $|x(t)| < 1$. The system is operating in the contact reaction mode when (2) is not satisfied or when $|x(t)| \geq 1$, i.e. when the output error evolves on and outside the performance bounds (Fig. 1). Let us denote with D the region of nominal performance operation, i.e. $D \triangleq (-1, 1)$ while $D^c = \Re \setminus D$ is the complement set of D.

Prescribed performance controllers do not allow the output error to escape the performance region guaranteeing prescribed performance and robustness to any external disturbance by utilizing a transformed error which is approaching infinity at the performance boundaries and is not defined outside the performance region. The control effort generated by a prescribed performance controller is increasing as the error approaches the boundary under the effect of a disturbance. The considerable stiffness induced by the prescribed performance control action may be undesired or even dangerous, if humans share the robot's workspace. Moreover, in practice, the output error may be forced outside the performance region due to the inability of the physical actuator to provide the demanded control effort or by not employing sufficiently high sampling rates. In such instances the control signal is not well defined and a prescribed performance controller may become potentially unsafe. In contrast to prescribed performance controllers, we propose controllers that allow the system solution to escape the nominal performance region under unintentional contacts.

B. Artificial potentials that induce vanishing vector fields

We proceed by defining a transformation $T(x)$ that is strictly increasing in x for $x \in D$ and saturated on and beyond the prescribed performance boundaries, i.e. for $x \in D^c$. Moreover, the transformation is concave in the first quadrant and convex in the third.

$$\begin{array}{ll} T : (-\infty, -1) \to -1, & \text{for } x(t) \leq -1 \ , \\ T : [-1, 1] \to [-1, 1], & \text{for } -1 \leq x(t) \leq 1 \ , \\ T : (1, \infty) \to 1, & \text{for } x(t) \geq 1 \ , \end{array} \tag{4}$$

satisfying the following properties:

$$\begin{array}{ll} T(0) = 0 \ , & \\ \partial T / \partial x > 0, & \forall x \in D \ , \\ \partial^2 T / \partial x^2 < 0, & \forall x : \ 0 < x < 1 \ , \\ \partial^2 T / \partial x^2 > 0, & \forall x : \ -1 < x < 0 \ . \end{array} \tag{5}$$

This transformation defines a smooth, nondecreasing, nonlinear, surjective mapping of the modulated error domain. The artificial potential induced by such transformations:

$$\mathcal{V}(x) = T^2(x) : \Re \to [0, 1] \tag{6}$$

is continuously differentiable, positive definite, i.e. $\mathcal{V}(x) > 0$ for $x \in \Re - \{0\}$, but it is not radially unbounded since regions $\mathcal{V}(x) \leq \beta$ are only closed for values of $\beta < 1$. Such potentials may allow a solution to escape the nominal performance region as opposed to the potentials induced by the transformations utilized in the prescribed performance controllers. A candidate transformation function is given in (7) with respective potential (8); it is illustrated in Fig. 2 and its potential is depicted in Figure 3. A fifth order polynomial could be an alternative transformation with higher degree of smoothness.

$$T(x) = \begin{cases} \sin\left(\frac{\pi}{2} x\right) & \text{for } -1 \leq x(t) \leq 1 \\ 1 & \text{for } x(t) > 1 \\ -1 & \text{for } x(t) < -1 \end{cases} . \tag{7}$$

$$\mathcal{V}(x) = \begin{cases} \frac{1}{2}(1 - \cos(\pi x)) & \text{for } x \in D \\ 1 & \text{for } x \notin D \end{cases} . \tag{8}$$

Let us further define the gradient of this potential field $\frac{1}{2} \nabla \mathcal{V}(x)$:

$$h(x) \triangleq \frac{\partial T}{\partial x} T(x) \ . \tag{9}$$

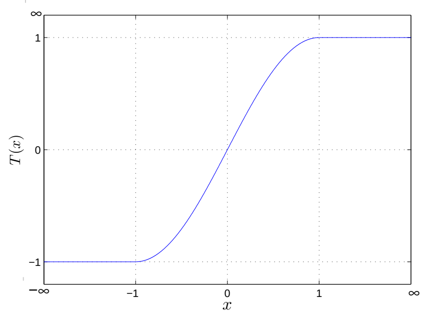

Fig. 2: A sinus based transformation function

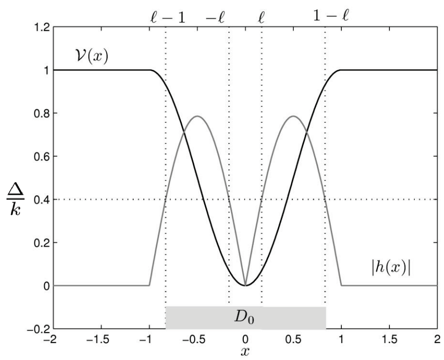

Fig. 3: The potential function $\mathcal{V}(x)$ (8), the invariant set (23) and the control term $|h(x)|$ (13)

Notice that $h(x)$ satisfies the following properties:

$$h(x) = 0 \text{ for } x \in D^c \ , \tag{10}$$

$$0 \leq xh(x) \leq c_h x^2 \ , \tag{11}$$

$$|h(x)| \leq h_M \ . \tag{12}$$

Potential $\mathcal{V}(x)$ is not typical in control design being unsuitable for global asymptotic stabilization and robustness analysis but allows the vanishing of the induced vector field without switching . For the transformation function (7):

$$h(x) = \begin{cases} \frac{\pi}{4}\sin(\pi x), & \text{for } x \in D \\ 0, & \text{for } x \notin D \end{cases} \ . \tag{13}$$

Function $h(x)$ lies in the first and third quadrant satisfying (11); its absolute value is shown in Fig. 3 yielding $h_M = \frac{\pi}{4}$.

C. Basic control signal

Using $h(x)$ (9) we can design a simple control input u for (1) as follows:

$$u = -\left[\alpha(t) + k_s\right] e - kh(x) \tag{14}$$

where k_s, k are positive control constants and $\alpha(t) \triangleq \frac{-\dot{\rho}(t)}{\rho(t)}$ is non-negative and bounded; for the exponential performance function, $\alpha(t)$ is, further, strictly decreasing with $0 < \alpha(t) \leq \alpha(0) < l$, $\lim_{t\to\infty}\alpha(t) = 0$.

Substituting control input (14) to the system (1) yields:

$$\dot{e} = -\left[\alpha(t) + k_s\right] e - kh(x) + d(t) \tag{15}$$

and taking into account that $\dot{x} = \frac{\dot{e} + \alpha(t) e}{\rho(t)}$ we get the closed loop system expressed with respect to the modulated error:

$$\dot{x} = \frac{1}{\rho(t)}\left[-kh(x) + d(t)\right] - k_s x \ . \tag{16}$$

For the unforced non-autonomous system (16), i.e. $d(t) = 0$, it is easy to establish that the origin $x = 0$ is a uniformly asymptotically stable equilibrium in $D \bigoplus D^c$. In case the error is forced outside the nominal performance region (2) by a significant contact force owing to a collision, the control term involving $h(x)$ vanishes due to property (10) while the remaining terms can be viewed as a proportional control action. The unforced closed loop system (16) becomes $\dot{x} = -k_s x$; hence x is drawn to ± 1 with a time constant $1/k_s$ that is, the error e is reaching the boundary of the prescribed performance region $\pm\rho(t)$. Given that no disturbance is acting at the system, x will return to the nominal operation mode ($|x| < 1$); hence e will cross the boundary converging uniformly and asymptotically to $e = 0$ ($x = 0$).

In the presence of a bounded input $d(t)$, the following theorem establishes the range of disturbances guaranteeing system operation in nominal mode:

Theorem 1: Consider a bounded disturbance input $|d(t)| < \Delta$ for the nonlinear system (16) such that:

$$\Delta \leq h_M k, \tag{17}$$

Then, there exists an invariant set $D_0 \subset D$ for the system state x; that is, initializing within D_0 guarantees a nominal performance error evolution in the sense of (2).

Proof: Using (6) for (16) the following can be satisfied in D: $\alpha_1(|x|) \leq \mathcal{V}(x) \leq \alpha_2(|x|)$ where α_1, α_2 are class $\mathcal{K}$ functions, and

$$\dot{\mathcal{V}}(x) = -\frac{2k}{\rho(t)}h^2(x) + \frac{2h(x)d(t)}{\rho(t)} - k_s\frac{\partial\mathcal{V}(x)}{\partial x}x \ , \tag{18}$$

$$\dot{\mathcal{V}}(x) \leq \frac{1}{\rho(t)}\left[-k|h(x)|^2 + \frac{|d(t)|^2}{k}\right] - k_s\frac{\partial\mathcal{V}(x)}{\partial x}x \ , \tag{19}$$

which implies that:

$$\dot{\mathcal{V}}(x) \leq -k_s\frac{\partial\mathcal{V}(x)}{\partial x}x, \ \text{ for } |h(x)| \geq \frac{\Delta}{k} \ . \tag{20}$$

Utilizing [14, Th. 4.18] we conclude the uniformly ultimate boundedness of the system state. In particular, we simplify the analysis by considering odd $h(x)$ functions although the analysis can be easily extended for the case of non-symmetric functions. If $h(x)$ is odd then, $|h(x)| = h(|x|)$ and it is now easier to calculate the domain of x wherein $h(|x|) \geq \frac{\Delta}{k}$. The equation $h(|x|) = \frac{\Delta}{k}$ can be solved with respect to $|x|$ if $\Delta \neq 0$ and (17) holds; the solution ς_1, ς_2 satisfy $0 < \varsigma_1 < \varsigma_2 < 1$ as shown in Fig. 3. We can then write

$$\dot{\mathcal{V}}(x) \leq -k_s\frac{\partial\mathcal{V}(x)}{\partial x}x, \quad \text{for} \quad \varsigma_1 \leq |x| \leq \varsigma_2 \ . \tag{21}$$

Hence defining

$$D_0 \triangleq \left\{x \in D : |x| \leq \varsigma_2\right\} \ , \tag{22}$$

if $x(0) \in D_0$ then $x(t) \in D_0 \subset D$, $\forall t \in R^+$ which implies that D_0 is invariant and the system remains in nominal performance operation. □

For the specific case of the candidate transformation function (7), the invariant set D_0 illustrated in Fig. 3 exists if $\Delta \leq \frac{\pi k}{4}$ according to condition (17) and is given by:

$$D_0 = \{x \in D : |x| \leq 1 - \ell\} \tag{23}$$

with

$$\ell = \frac{1}{\pi}\arcsin(\frac{4\Delta}{\pi k}) \ . \tag{24}$$

Remark 1: The maximum disturbance allowing a nominal performance operation mode (17) can be regulated by the control design constant k. Moreover, for nominal performance operation, constant ρ_0 of the performance function should be selected such that (22) is satisfied at $t = 0$, i.e., $\rho_0 \geq \frac{e(0)}{\varsigma_2}$.

Remark 2: When the system operates in the contact reaction mode the closed loop system $\dot{e} = -[\alpha(t) + k_s]e + d(t)$ or $\dot{x} = -k_s x + \frac{d(t)}{\rho(t)}$ is ISS (input-to-state stable) for the disturbance input $d(t)$ since $\alpha(t) + k_s \geq k_s > 0$. If t_e is the time instant the disturbance vanishes, the system will return to the nominal operation in $\frac{\ln x(t_e)}{k_s}$ s.

Since our objective is a robot control design which complies with large disturbances, there is no need of choosing high values for k_s in order to shrink the ultimate bound of the system given by $\frac{d(t)}{k_s}$ (since $\lim_{t\to\infty} \alpha(t) = 0$).

III. The Proposed Robot Controller

Consider a n DOF robotic manipulator with $q \in \Re^n$ denoting its joint position vector and $p_e \in \Re^3$, $R_e \in SO(3)$ describing the position and the orientation of the end-effector with respect to the inertial frame respectively. Furthermore, consider a set of generalized coordinates $p = [p_e^T \quad \phi_e^T]^T$ where ϕ_e are orientation parameters e.g Euler angles. Notice that $\dot{p} = J_A(q)\dot{q}$ with $J_A(q)$ being the analytical Jacobian while the robot Jacobian $J(q)$ maps joint velocities to the end-effector generalized velocity $v = J(q)\dot{q}$ with $v \triangleq [\dot{p}_e \ \omega_e]^T \in \Re^6$. Under the assumption of a non-redundant manipulator operating in a region in which J_A is nonsingular, the robot dynamic model can be represented with respect to p in the operational space as follows:

$$M_p(p)\ddot{p} + C_p(p, \dot{p})\dot{p} + G_p(q) + W(p)F_c = u_p \ , \tag{25}$$

where F_c is a bounded disturbance typically arising by unforseen contacts of the arm with a human or the environment (in general, $F_c \in \Re^6$) and

$$M_p(p) = \left[J_A(q)H^{-1}(q)J_A^T(q)\right]^{-1}, \tag{26}$$

$$C_p(p,\dot{p})\dot{p} = J_A^{-T}(q)C(q,\dot{q})\dot{q} - M_p(p)\dot{J}_A(q)\dot{q} \tag{27}$$

$$G_p(p) = J_A^{-T}(q)G(q), \tag{28}$$

$$W(p) = J_A^{-T}(q)J^T(q), \tag{29}$$

$$u_p = J_A^{-T}(q)u. \tag{30}$$

with $H(q)$ being the positive definite robot inertia matrix, $C(q, \dot{q})\dot{q}$ the Coriolis and centripetal force and $G(q)$ the gravity force vector.

For simplicity and without loss of generality we proceed by considering the non-redundant position tracking problem; in such a case $W(p) = I_3$ where I_3 is the unit matrix with dimension 3. Our objective is to design a state feedback control law, in order to force the robot's end-effector position $p_e(t)$ to track a given desired trajectory $p_d(t)$ with prescribed performance under its nominal operation mode in the sense of confining the evolution of each position error coordinate $e_i = p_{ei}(t) - p_{di}(t)$ within a predefined region that is bounded by $\pm\rho_i(t)$ under small disturbances $F_c \in \Re^3$ and to enable a smooth compliant reaction outside the performance region when the contact force magnitude exceeds an allowed level, returning to the nominal mode after the disturbance vanishes.

Theorem 2: Consider the model of a robotic manipulator (25) in the operational space, the desired position trajectory $p_d(t) \in \Re^3$ and performance functions $\rho_i(t)$ $i = 1, \ldots, 3$ as defined in (3) that incorporate the desired performance bounds of the position tracking error elements $e_i(t)$ in the nominal operation mode as well as transformations $T_i(x_i)$ as in (7) for the modulated error elements $x_i = \frac{e_i}{\rho_i}$. Moreover, define the intermediate control signal $\dot{p}_r \in \Re^3$ with elements:

$$\dot{p}_{ri} = \dot{p}_{di} - [\alpha_i(t) + k_{si}]e_i(t) - k_i h_i(x_i) \ , \tag{31}$$

where k_i, k_{si} are positive control constants, $h_i(x_i)$ is defined as in (9) and $\alpha_i(t) = \frac{-\dot{\rho}_i(t)}{\rho_i(t)}$. Assuming a robot motion away from singular positions, the passivity model-based control law:

$$u_p = -K_v s + M_p(p)\ddot{p}_r + C_p(p, \dot{p})\dot{p}_r + G_p(p) \ , \tag{32}$$

where $s = \dot{p} - \dot{p}_r$ and K_v is a diagonal matrix of positive control parameters achieves prescribed performance (2) for the nominal operation and compliance under disturbances in contact reaction returning to nominal operation after the disturbance vanishes.

Proof: Substituting (32) in (25) we obtain the closed loop system:

$$M_p(p)\dot{s} + (C_p(p, \dot{p}) + K_v)s + F_c = 0 \ . \tag{33}$$

Consider now the positive definite radially unbounded function:

$$\mathcal{L} = \frac{1}{2}s^T M_p(p)s \ , \tag{34}$$

which satisfies the following inequality

$$\frac{\lambda_m}{2}\|s\|^2 \leq \mathcal{L} \leq \frac{\lambda_M}{2}\|s\|^2 \ , \tag{35}$$

where λ_m, λ_M are positive constants related to the robot's minimum and maximum eigenvalue of $M_p(p)$. Differentiating (34) with respect to time along the solutions of (33) and taking into account the skew symmetry of $\dot{M}_p(p) - C_p(p, \dot{p})$, we obtain:

$$\dot{\mathcal{L}} = -s^T K_v s - s^T F_c(t) \ . \tag{36}$$

Let k_v be the minimum entry of K_v; then, $\dot{\mathcal{L}}$ can be upper bounded as follows:

$$\dot{\mathcal{L}} \leq -k_v\|s\|^2 + \|s\|\|\tau_d(t)\| \ , \tag{37}$$

$$\dot{\mathcal{L}} \leq -\frac{1}{2}k_v\|s\|^2 - \frac{1}{2}k_v\|s\|\left(\|s\| - \frac{2\|F_c(t)\|}{k_v}\right) \ . \tag{38}$$

Defining the region $B = \{s \in \Re^3 : \ \ \|s\| \leq \frac{2\|F_c(t)\|}{k_v}\}$ it is clear that:

$$\dot{\mathcal{L}} \leq -\frac{1}{2}k_v\|s\|^2, \quad \text{for } s \notin B \ \ , \tag{39}$$

which proves the uniform ultimately boundedness of s. In fact, using (35) and (39) it can be shown that

$$\|s\| \leq \sqrt{\frac{\lambda_M}{\lambda_m}}\|s(0)\|e^{-(\frac{k_v}{2\lambda_M})t}, \quad \text{for } s \notin B \ \ , \tag{40}$$

$$\|s\| \leq \sqrt{\frac{\lambda_M}{\lambda_m}}\frac{2\|F_c(t)\|}{k_v}, \quad \text{for } s \in B \ \ , \tag{41}$$

which can be combined in the following:

$$\|s\| \leq \max\sqrt{\frac{\lambda_M}{\lambda_m}}\left(\|s(0)\|e^{-(\frac{k_v}{2\lambda_M})t}, \frac{2\|F_c(t)\|}{k_v}\right) \ \ , \tag{42}$$

demonstrating an input-to-state stability, [15], for the pair $F_c(t)$, s of (33).

Substituting (31) in the definition of s ($s = \dot{p} - \dot{p}_r$), yields $s = \dot{e} + (A(t) + K_s)e(t) + KH(x)$ where $A(t)$, K_s, K are diagonal matrices with entries $\alpha_i(t)$ and $k_{si}, \ k_i > 0$ respectively and $H(x)$ is a vector with elements $h_i(x_i)$. Given (42), s is bounded for a bounded disturbance $F_c(t)$ and we can therefore obtain the following disturbed error system:

$$\dot{e}(t) = -\left[A(t) + K_s\right]e(t) - KH(x) + s(t) \ . \tag{43}$$

Each element of (43) is related to the error scalar system (15) having as disturbance input the ith element of $s(t)$. Let us take the example of a contact force F_c and system (33) operating at the steady state, i.e. $e^{-(\frac{k_v}{2\lambda_M})t} \simeq 0$. From (42) we observe that $\|s(t)\| \sim \|F_c\|/k_v$ remains the main source of disturbance at the velocity control level. Following Theorem 1 for each element of (43) shows that the controller guarantees that the system, operating at steady state, stays in nominal performance operation for disturbances up to a tunable threshold (reflecting modeling errors), while escaping this mode for higher disturbances as are those arising from unintentional contacts. In this case the robot reaction is stable and compliant, returning to the nominal operation mode after the disturbance vanishes. □

Remark 3: Notice that when the system operates in the contact reaction mode where $h_i(x_i) = 0$ and $\alpha_i(t) \simeq 0$ (in the steady state region of the performance function), the reference velocity (31) becomes $v_{ri} = \dot{p}_{di} - k_{si}e_i$ and the model-based controller (32) guarantees system stability.

Remark 4: A brief unexpected contact is so far assumed in the presentation of the proposed control law. However, if the contact persists contact forces will keep increasing even with low output stiffness since the reference position advances. In that case a post-contact strategy that abandons the desired trajectory is mandatory so that tracking errors cease to build up.

IV. Simulation Results

We consider a 3 DOF spatial robotic manipulator with rotational joints, link lengths $l_1 = 0$, $l_2 = l_3 = 0.5$ m, link masses $m_1 = m_2 = m_3 = 1$ Kg and inertias $I_{x1} = I_{y1} = 0$, $I_{z1} = I_{x2} = I_{x3} = 4.15 \times 10^{-4}$, $I_{y2} = I_{z2} = 2.1 \times 10^{-2}$, $I_{y3} = I_{z3} = 0.39 \times 10^{-2}$ Kg m^2. The robot is initially at rest at the position $p_e(0) = [0.55 \ 0.55 \ 0.55]^T$(m) and is desired to move to the target location $p_{df} = [0.249 \ 0.249 \ 0.249]^T$(m) for a duration of $T = 3$ sec, following a fifth-order polynomial trajectory for each position coordinate: $p_d(t) = p_{do} + (p_{df} - p_{do})\big(10(\frac{t}{T})^3 - 15(\frac{t}{T})^4 + 6(\frac{t}{T})^5\big)$ where $p_{do} = [0.549 \ 0.549 \ 0.549]^T$(m) is the desired trajectory's initial position resulting in an initial position error of $e(0) = [0.001 \ 0.001 \ 0.001]^T$(m). The performance function is defined as in (3) and considered to be the same for all position errors $e_i(t)$ $i = 1, 2, 3$, with $\rho_{i0} = 0.02$ set high enough to ensure initialization within the invariant set D_0 for a range of disturbance magnitudes, $\rho_{i\infty} = 10^{-3}$ corresponding to an accuracy of 1 mm and $l_i = 20$ for a fast transient response. Control constants from (32), (31) are set to: $K_v = 30I_3$, with I_3 being the identity matrix of dimension 3, $k_{si} = 5$ and $k_i = 0.4$, $i = 1, 2, 3$.

We initially consider a contact force $F_c(t)$ applied to the robot's end-effector along the x Cartesian direction in the form of a smooth pulse simulated by the function: $F_c(t) = \frac{F_E}{2}(\ \tanh(100(t - 0.95)) - \tanh(100(t - 1.05))\ )$ N where F_E is the pulse amplitude in order to evaluate the robot's reaction to a disturbance from the point of view of an apparent output stiffness (K_{stiff}) via a series of simulations with contact forces of various amplitudes. The stiffness values are calculated by the ratio of the pulse amplitude F_E to the maximum error displacement. Results from two simulation cases in nominal and contact reaction modes with $F_E = 10$ N and $F_E = 40$ respectively are shown in Fig 4, 5 depicting positions error responses and the respective contact force while Fig. 6 depicts the calculated stiffness for all tested cases. Notice that two distinct areas of stiffness values (K_{stiff}) corresponding to the nominal performance and contact reaction modes (Fig. 6) are revealed. In the contact reaction mode the output stiffness is characterized by low values (focused subplot) enhancing human and robot safety as compared to the high stiffness values in the nominal performance operation ensuring robustness to model errors and small disturbances. With the specific gain selection it is clear that a nominal performance is achieved for disturbances up to approximately 12 N. This level can be regulated by changing the value of k_i.

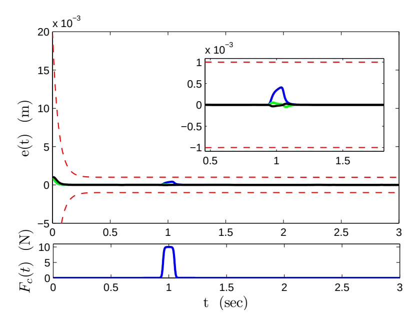

Fig. 4: Cartesian position error component responses in nominal performance mode.

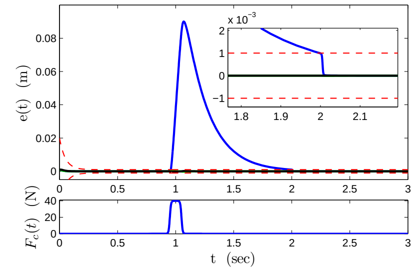

Fig. 5: Cartesian position error component responses in contact reaction mode.

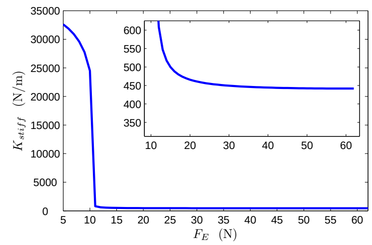

Fig. 6: Estimated output stiffness

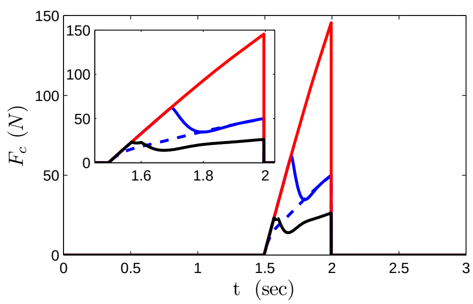

Fig. 7: Interaction forces. Black solid line - proposed law. Red solid line - high stiffness impedance controller. Switched impedance controller: Blue solid line - 0.2 sec delay. Blue dashed line - 0.001 sec delay.

Moreover, we have simulated the case of a contact with an environment modeled as a spring with stiffness of 1000 N/m, obstructing the motion of the arm for 0.5 sec. For comparison purposes we have simulated the case of the robot being under an impedance control scheme given by the following closed loop $M_p(p)\ddot{e} + (C_p(p, \dot{p}) + K_d)\dot{e} + K_p e + F_c(t) = 0$. We switch impedances between the values $K_p = 24500 I_3$, $K_d = 450 I_3$ and $K_p = 400 I_3$, $K_d = 50 I_3$ with a delay of 0.001 sec and 0.2 sec after contact accounting for various delays due to the contact detection and reaction response. Figure 7 displays the interaction forces developed during the unintentional contact. Notice the significant contact forces that develop with the high stiffness impedance controller and when there is a switching delay. The proposed controller by smoothly traversing the nominal and contact reaction regions, outperforms all impedance control cases, achieving the lowest interaction forces and hence enhanced human and robot safety. As regards tracking performance in the nominal operation mode, both the highest stiffness impedance and the proposed controller perfectly track the desired trajectory having negligible differences. However, when a contact with a human is expected the closed loop system under impedance control should be made more compliant (than the highest stiffness considered herein) in order to keep the interaction forces limited [16].

V. Conclusions

This work proposes a control law that achieves prescribed tracking performance of a desired end-effector trajectory in its nominal operation under disturbances up to a tunable threshold and a smooth compliant reaction outside the performance region when this threshold is exceeded, eventually returning to the nominal mode after the force vanishes. Simulations demonstrate the quality of performance and enhanced safety achieved by the proposed controller under unintentional contact, outperforming a switching impedance scheme.

References

[1] J. Heinzmann and A. Zelinsky, "Quantitative safety guarantees for physical human-robot interaction," *The International Journal of Robotics Research*, vol. 22, no. 7-8, pp. 479–504, 2003.

[2] J. Choi, S. Park, W. Lee, and S. C. Kang, "Design of a robot joint with variable stiffness," in *Proceedings - IEEE International Conference on Robotics and Automation*, 2008, pp. 1760–1765.

[3] S. Wolf and G. Hirzinger, "A new variable stiffness design: Matching requirements of the next robot generation," in *Proceedings - IEEE International Conference on Robotics and Automation*, 2008, pp. 1741–1746.

[4] N. G. Tsagarakis, I. Sardellitti, and D. G. Caldwell, "A new variable stiffness actuator (CompAct-VSA): Design and modelling," in *IEEE International Conference on Intelligent Robots and Systems*, 2011, pp. 378–383.

[5] A. Albu-Schaeffer, C. Ott, and F. Petit, "Energy shaping control for a class of underactuated Euler-Langrange systems," in *Proc. 10th Intern. IFAC Symposium on Robot Control*, 2012, pp. 567–574.

[6] A. De Luca and R. Mattone, "Sensorless robot collision detection and hybrid force/motion control," in *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*, 2005, pp. 999–1004.

[7] A. De Luca, A. Albu-Schaffer, S. Haddadin, and G. Hirzinger, "Collision detection and safe reaction with the dlr-iii lightweight manipulator arm," in *Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2006, pp. 1623–1630.

[8] E. Magrini, F. Flacco, and A. De Luca, "Control of generalized contact motion and force in physical human-robot interaction," in *IEEE International Conference on Robotics and Automation (ICRA)*, 2015, pp. 2298–2304.

[9] S. Oh, H. Woo, and K. Kong, "Frequency-shaped impedance control for safe human-robot interaction in reference tracking application," *IEEE/ASME Transactions On Mechatronics*, vol. 19, no. 6, pp. 1907–1916, 2014.

[10] S. Golz, C. Osendorfer, and S. Haddadin, "Using tactile sensation for learning contact knowledge: Discriminate collision from physical interaction," in *IEEE International Conference on Robotics and Automation (ICRA)*, vol. 11, 2015.

[11] D. Lee and C. Ott, "Incremental kinesthetic teaching of motion primitives using the motion refinement tube," *Auton Robot*, vol. 11, pp. 115–131, 2011.

[12] C. P. Bechlioulis and G. A. Rovithakis, "Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance," *IEEE Transactions on Automatic Control*, vol. 53, no. 9, pp. 2090–2099, 2008.

[13] Y. Karayiannidis and Z. Doulgeri, "Model-free robot joint position regulation and tracking with prescribed performance guarantees," *Robotics and Autonomous Systems*, vol. 60, no. 2, pp. 214–226, 2012.

[14] H. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2002.

[15] H. Marquez, *Nonlinear Control Systems*. Wiley-Interscience, 2003.

[16] B. Siciliano, L. Sciavicco, and L. Villani, *Robotics: Modelling, Planning and Control*, ser. Advanced Textbooks in Control and Signal Processing. Springer, 2009.